

Towards Sustainable Buildings: Novel Strategies for the Design of Vibration Resistant Cross-Laminated Timber Floors

WP3 – Numerical simulations
and metamodeling

Report on conducted numerical simulations



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1. Summary

The development of computational software tools and utilities is one of the most important activities of the Substrate4CLT project. As indicated in the project proposal, several team members have already been dealing with cutting-edge simulation tools based on dynamic stiffness and finite element methods in conjunction with different laminated plate and beam theories. Before the project started, some methods were implemented by the team members within the original computational tools [1, 2]. This served as a basis for the development of new tools and utilities within the Substrate4CLT.

The development of computational tools was completed within the scope of WP3 - Numerical simulations and metamodeling. After the initial setup of necessary tools, their development and calibration continued during the project implementation. The above activities have been elaborated in the Report on Upgraded Computational Tools, which has been published at the project's website at the following [link](#). The development has been divided in the following areas, based on their purpose:

- **calculation of mechanical properties of inter-panel connections in CLT.** The actual connections are commonly neglected in design calculations, thus a CLT floor is modelled either as a monolith slab or more frequently as a set of CLT panels with no connections. The project proved that including the inter-panel connections significantly influences the dynamic behavior of CLT floors due to pedestrian-induced loading (see [3] for details), and provided calculation sheets and MATLAB [4] scripts for the calculation of connection properties;
- **dynamic response of CLT floors.** A Python-based [5] computational tool named Hindu has been developed to obtain the dynamic response of CLT floors due to human activities (walking or jumping). Hindu calculates the CLT floor's response by modal superposition technique and accounts for spatial variation of the human dynamic force. The process consists of several steps following the framework adopted in [6]. The software user can choose between several implemented force models so far, that are commercial and available in the literature [7-10]. Hindu is shared with the user community through the GitHub repository https://github.com/mmilojevicgrf/HINDU_software.
- **static analysis of multilayer beam- and plate-like structures.** The object-oriented computational framework FLWTFEM [2] is primarily written for the analysis of laminated composite plates using layerwise plate theory [11], and implies the static and dynamic (free vibration) analysis of multilayered plate-like structures of arbitrary geometry. The FLWTFEM has been constantly improved by the authors for academic purposes during the project implementation. Within a Substrate4CLT project, FLWTFEM is extended by adding new analysis options: multi-modal analysis of multilayer plates [12], and progressive failure analysis of multilayer structures, using the smeared-crack-band model [13]. It is implemented in Matlab and shared with the user community through the GitHub repository www.github.com/miregrf/FLWTFEM.

The developed software tools, along with the commercial software [14], have been used to conduct a number of numerical simulations during the project cycle. The numerical simulations have been performed to support experimental analysis (both static and dynamic), within the Work Packages 2, 3 and 4.



2. Mechanical properties of inter-panel connections

2.1 Introduction

Long-span cross-laminated timber (CLT) floors are typically an assembly of prefabricated CLT panels connected together on the site. The actual connections are commonly neglected in design calculations. Hence, a CLT floor is modelled either as a monolith slab or more frequently as a set of CLT panels with no connections at all.

Although the inter-panel connections are relatively complex in reality, they are typically modelled as an equivalent 2D elastic strip between the CLT panels [15, 16]. This relatively simple yet robust model can be used with ease in design practice, regardless the finite element (FE) software used to extract vibration modes of a CLT floor.

Modal properties of a floor are the key structural parameters needed for its VSA. Given that timber is an orthotropic material having three principal material axes, explicit formulas based on isotropic plate representation cannot reliably provide natural frequencies apart from the fundamental [17]. Consequently, in engineering practice CLT structures are commonly modelled using the finite element (FE) method. Due to their inherent multi-layer and multi-panel features, aided by a great level of uncertainty of boundary conditions and modal damping, numerical modelling of CLT floors is a fairly challenging task. A FE model that is not too complex yet accurate enough description of the reality is crucial for a reliable VS assessment.

The Substrate4CLT project provided a comprehensive numerical study [3] designed to examine the influence of two most common inter-panel connections, i.e. single surface spline and half-lapped joint, on vibration modes and vibration responses of a range of different CLT floors due to pedestrian-induced loading.

The first set of simulations has been performed on two floor layouts: 6x6m floor (Figure 1, left) and 4x4m floor (Figure 1, right), both composed from 2 inter-connected panels. Floors were modelled in Abaqus CAE using S4R finite elements (conventional 4-node shell element with reduced integration). The mesh size was 0.05m. CLT cross section was modelled with composite section, while connections were modelled as equivalent elastic strip with homogeneous cross section. Elastic strip was rigidly connected to adjacent panels.

Modal properties were calculated for 16 different scenarios in total.

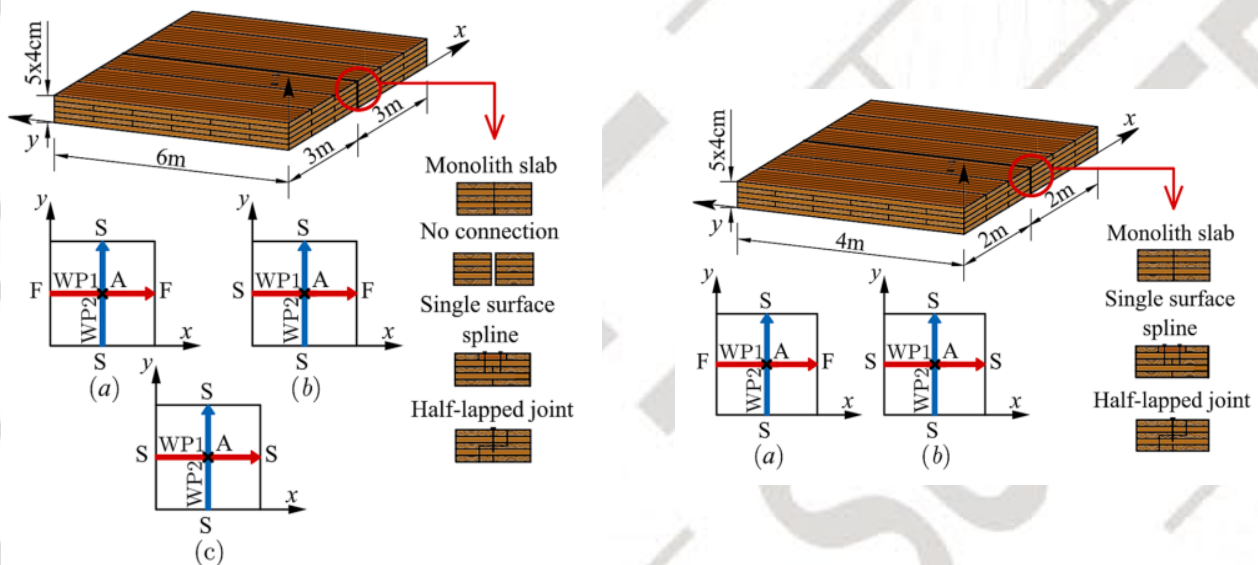


Figure 1. Floor layouts investigated in [3]



Several connection types and boundary conditions (Figure 2) were analyzed.

Example	Floor Type	Boundary conditions	Connection
Example 1	LFF	SFSF	monolith slab
			no connection
			single surface spline
		SSSF	half-lapped joint
			monolith slab
			single surface spline
		SSSS	half-lapped joint
			monolith slab
			single surface spline
Example 2	HFF	SFSF	half-lapped joint
			monolith slab
			single surface spline
		SSSS	half-lapped joint
			monolith slab
			single surface spline

Note: S – simply-supported edge; F – free edge.

Figure 2. Connection types, boundary conditions and frequency levels investigated in [3]

2.2 Global sensitivity analysis (GSA)

In another comprehensive study conducted by the Substrate4CLT team and presented in [18], the experimental program, performed through WP4 of the project, was designed to provide fundamental data needed to determine and verify parameters of the elastic strip representation of half-lapped joints in real CLT floors. This was done by step-by-step fitting measured modal properties of floors composed of one and two CLT panels to their FE models. Both floors were placed on elastic supports yielding the first several rigid body modes of vibration. Rigid body modes of the single-panel floor were used to update modelling parameters of the supports alone. Deformation modes (i.e. bending and torsional) of a single-panel floor were used to update timber material properties, while the properties of the elastic strip were updated using both rigid body and deformation modes of the two-panel floor. Finally, the results of the finite element model updating (FEMU) were verified on the measured modal properties of the three-panel floor screwed along the shorter edges to a pair of glue-laminated timber (GLT) beams. Layout for all three floors is presented in Figure 3, while three-dimensional FE models created in Abaqus CAE are shown in Figure 4. 3D numerical models were formed using C3D20R finite elements (20-node quadratic brick element, with reduced integration) with size 0.1m. Each layer of the CLT floor slab was modelled as an individual lamina with orthotropic material properties. Since a real CLT panel consists of cross-wise layers, each lamina in the numerical models was oriented 90° relative to the adjacent laminae. The bond between the layers was assumed perfectly rigid, with the inter-connection modelled via the tie-constraints. Connections were modelled as equivalent elastic strip with homogeneous cross section, rigidly connected to adjacent panels.

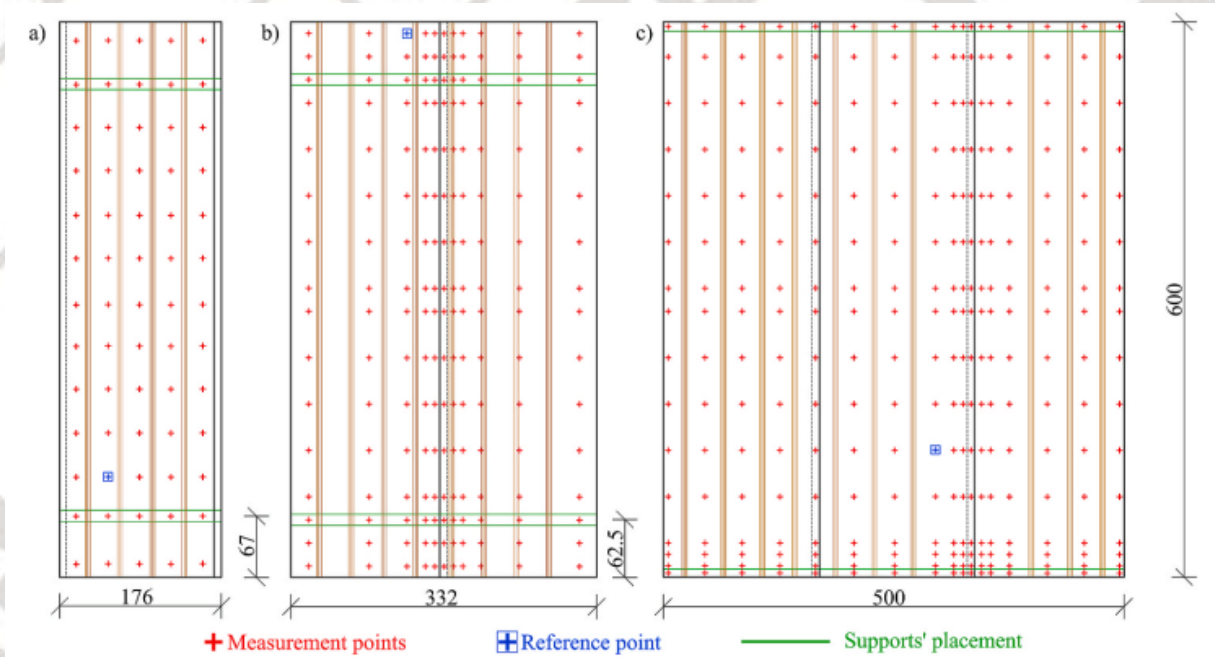


Figure 3. Dimensions and testing grid for a) single-panel, b) two-panel and c) three-panel floor [18]

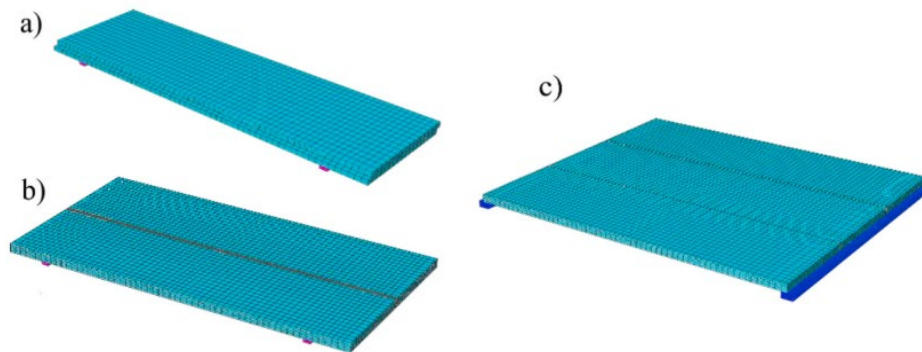


Figure 4. Three-dimensional numerical models of a) single-panel, b) two-panel and c) three-panel floor [18]

Two GSAs based on the Spearman's correlation coefficients (Figures 5 and 6) were carried out independently. The first studied the material properties of the elastic supports and the selected C24 timber properties of the single-panel floor, while the second focused on the geometric and material properties of the elastic strip. The first GSA revealed that modal properties of the single-panel floor are not affected by the elasticity modulus E_R , Poisson's coefficients ν_{LR} and ν_{TR} , shear modulus G_{LR} and elastic support density ρ_{sup} . Elasticity modulus of the elastic support E_{sup} and Poisson's coefficient ν_{sup} significantly affect the natural frequency and mode shape of the third rigid body mode as well as MACs of the several deformation modes. However, there is no influence on their natural frequencies. Transverse elasticity modulus E_T and Poisson's coefficient ν_{LT} affect only higher mode shapes. The most influential parameters are the longitudinal elasticity modulus E_L and shear moduli G_{LT} and G_{TR} .

The second GSA showed that width a_{conn} is the most influential property of the elastic strip. Shear modulus G_{conn} affects both natural frequencies and mode shapes of modes 1, 4, 7, 9, 10, 12 and 14, but they are not sensitive to variations of the elasticity modulus E_{conn} and Poisson's ratio ν_{conn} . On the other hand, natural frequencies of the RBM and modes 2, 5, 15, and 16 are influenced by E_{conn} and ν_{conn} , but unaffected by G_{conn} .

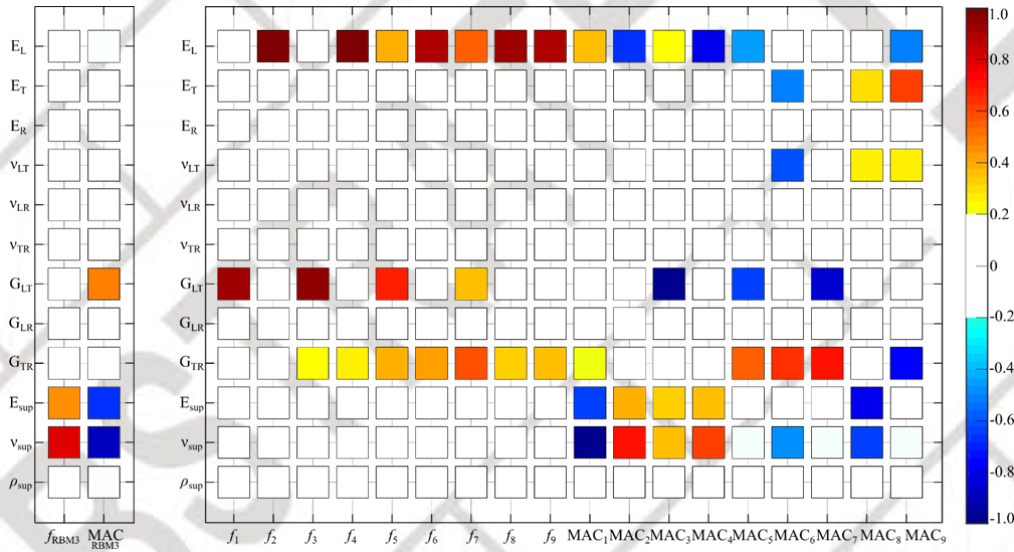


Figure 5: Spearman's correlation coefficients between timber and elastic support material properties, and single-panel floor's numerical frequencies and MAC values [18]

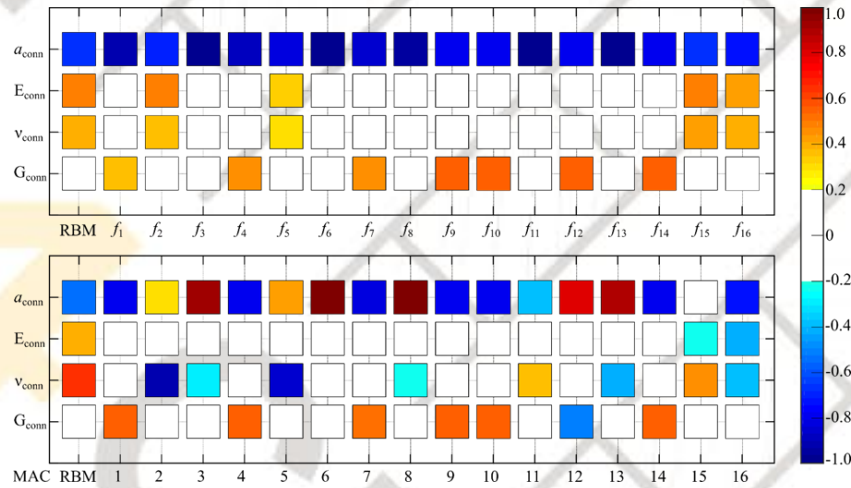


Figure 6: Spearman's correlation coefficients between connection properties and two-panel floor's numerical natural frequencies (up) and MAC values (down) [18]

For each GSA, 500 simulations were performed (1000 simulations in total).

2.3 Finite element model updating (FEMU)

Generally speaking, FEMU is an optimization problem, aiming to find the values of the selected modelling parameters which minimize the difference between the experimental and numerical behaviour of the structure. In modal analysis, the difference is typically quantified in terms of frequency (r_f) and mode shapes (r_m) residuals.

Within the study presented in [18], a single-panel floor was used to fit material properties of the timber and the boundary conditions. Then, the FEMU of two-panel floor yielded the actual values of the strip modelling parameters. Finally, these were successfully verified by direct comparison between numerically calculated and experimentally measured modal properties of a full-scale model of a simply supported three-panel floor.

Harmony Search Algorithm [19] (HSA)-based optimization was carried out using Python scripting in conjunction with Abaqus CAE.

The GSAs outputs helped to design a step-by-step strategy for FEMU of the single-panel and two-panel floors (Figure 7). Values of non-sensitive parameters E_R , v_{LR} , v_{TR} , G_{LR} and ρ_{sup} were set to their initial values. Sensitive



modelling parameters of both the elastic support and timber material were updated using the experimentally measured modal properties of the single-panel floor. The initial values of E_{sup} and v_{sup} were updated based on the natural frequency and MAC value of the RBM 3. Finally, E_L was updated based on the modal properties of the second deformation mode, while E_T , G_{LT} , G_{TR} , and v_{LT} were updated using deformation mode 1 and modes 3-9.

In the second step, the numerical model of the two-panel floor was updated using the optimised parameters of a single-panel floor from the first step. This became the initial model for the FEMU featuring optimization of the elastic strip parameters. The FEMU was carried out featuring all four parameters simultaneously.

Finally, in the third step the parameters updated through the first two steps were validated against the experimental results of the three-panel floor.

Each optimization phase started with 50 initial simulations. In each new iteration, 25 new simulations were performed. The optimization would terminate after 100 iterations. For one optimization that is $50+25 \times 100=2550$ simulation. There were four optimization phases conducted (see Figure 7), which counts **10200 simulations**.

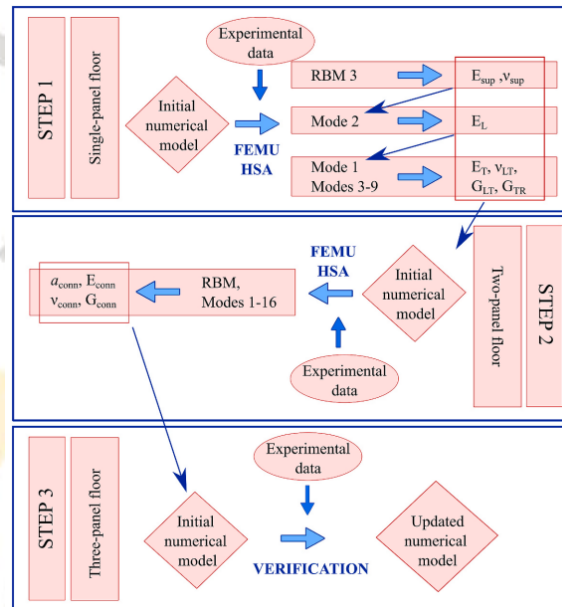


Figure 7. Algorithm to optimize FE modelling parameters [18]

Detailed 3D FE modelling is not typical in floor design. Structural designers normally use two-dimensional (2D) FE floor models as they are simpler and can be created faster. To examine whether the 2D model can provide results as good as the 3D model, the three-panel floor was re-modelled in Abaqus CAE using S4R finite element (Figure 8). Simply supported boundary conditions were applied along the two parallel edges as illustrated in Figure 8. The mesh size was 0.1m. The model was made using previously updated timber and connection properties. The analysis showed that , the 2D and 3D numerical models represent equally well the experimental results.

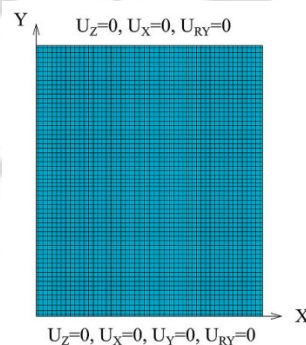


Figure 2. Two-dimensional numerical model of three-panel floor



3. Dynamic response of CLT floors

Simulations of floor vibrations, presented in [3], use the models of the vertical walking loading reported in Arup's design guide for pedestrian-induced vibrations [9]. Here, the cut-off frequency between the low-frequency floors (LFF) and high-frequency floors (HFF) is 10.5 Hz. The need to distinguish between LFFs and HFFs is rather artificial and driven by the lack of available walking force models that can describe the full amplitude spectrum of the measured vertical force signals. Hence, the two floor types have different mathematical characterisations of the walking loading.

The walking force model pertinent to LFF is a sum of four harmonics described by a Fourier series. The frequency of each harmonic is an integer multiple (i.e. 1-4) of the selected walking frequency (also called footfall rate) in the range 1.5-2.5Hz. The worst-case response scenario is the floor resonance with one force harmonic. On the other hand, walking loading for HFFs is a series of vertical impulses corresponding to each footfall. Vibration response between two successive footfalls has a transient character and dies out quickly due to typically large damping of HFFs.

Based on the modal superposition method [20], the guideline provides the estimates of the peak and RMS vibration responses corresponding to the steady state oscillations at a selected point on the floor surface due to a stationary modal walking force. The force is placed typically at the least favourable point on the floor, i.e. the peak of a targeted mode shape.

To get a closer insight into pedestrian-induced vibrations of CLT floors and in particular what happens around the connection line between the panels, [3] presents response time histories for selected straight walking paths (see Figure 1). The guideline recommends that in the case of LFFs, modes up to 15 Hz should be considered in the modal superposition. The present study reports all modes up to 30 Hz. This higher frequency limit is in line with the renowned experimental measurements and numerical studies on floor vibrations that used actual records of walking force time histories [10]. In the case of HFFs, all modes with frequencies less than twice the fundamental frequency were taken into account, as suggested in [9].

Two walking paths for each example were analyzed (WP1 and WP2 in Figure 1). Previously calculated modal properties (in Abaqus CAE) were fed into Python-based Hindu software, and response in the floor's midpoint for each scenario was calculated. According to Figure 2, 16 different scenarios were considered, which, multiplied by 2 walking paths for each scenario, results in 32 simulations in total.

Similar analysis was carried out in [21] on complex-shaped CLT floor, presented in Figure 9. Floor was modelled in Abaqus CAE using S4R finite elements (conventional 4-node shell element with reduced integration). The mesh size was 0.1m. CLT cross section was modelled with composite section, while connections were modelled as equivalent elastic strip with homogeneous cross section. Elastic strip was rigidly connected to adjacent panels.

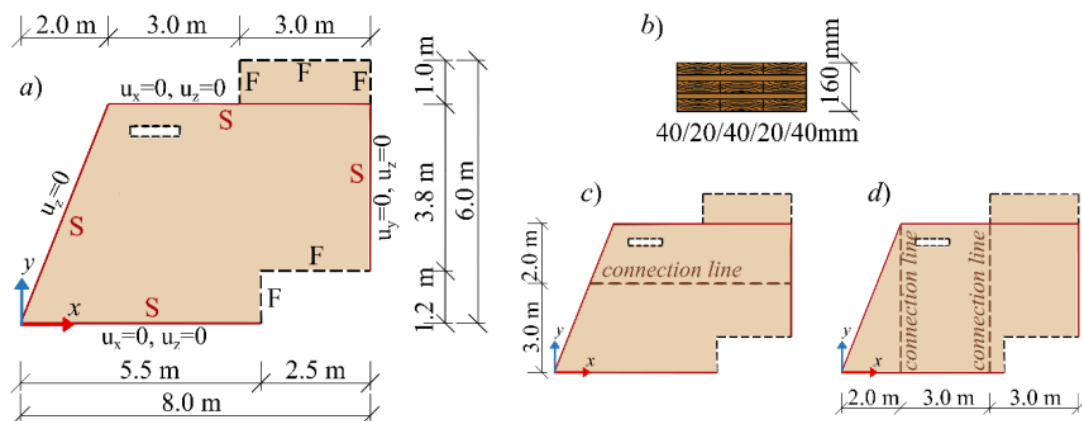


Figure 9. Complex-shape CLT floor structure (a) floor layout with boundary conditions, (b) floor cross-section, (c) floor configuration with panels oriented in x and (d) y direction [21]



Two different scenarios were studied – panels oriented in x (Figure 9c) and panels oriented in y direction (Figure 9d). For each scenario, monolithic floor was analyzed as well for comparison. That is **4 simulations** in total for calculation of modal properties.

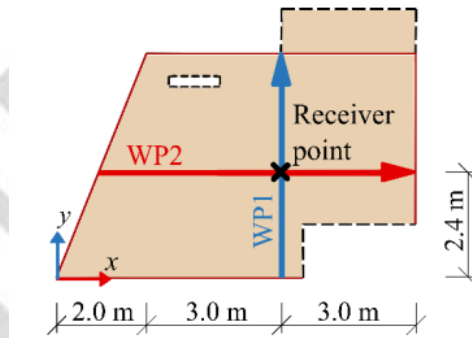


Figure 10. Two walking paths and receiver point [21]

Two walking paths for each example were analyzed (Figure 10). Previously calculated modal properties (in Abaqus CAE) were fed into Python-based Hindu software, and response in the receiver point for each scenario was calculated – **8 simulations** in total.



4. Static analysis of multilayer beam- like structures

Static analysis of beam-like CLT and CLT-FRP specimens, performed within the WP2 of the Substrate4CLT project, has been supported by the extensive numerical simulations conducted using both the commercial and original software.

4.1 Simulations using the FEM-based commercial software

In order to better understand stress-strain behaviour of CLT and CLT45 panels, a relatively simple numerical model was developed using finite element software Abaqus CAE. Geometry of the panels, loading and boundary conditions were modelled so as to correspond to the experimental set-up (see Figure 11).

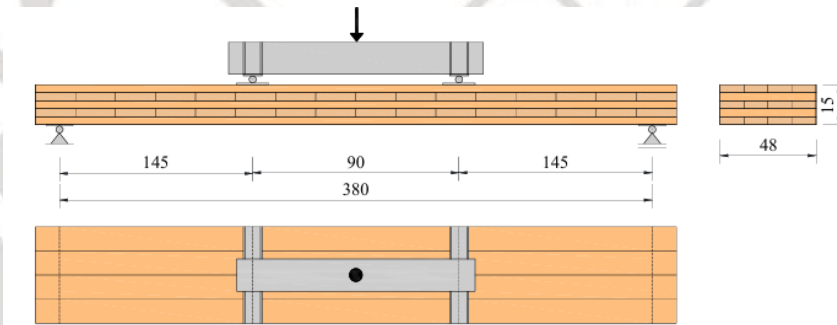


Figure 11. Experimental setup [21]

The CLT panels were modelled using S4R finite elements (4-node shell finite elements with reduced integration), with element size of 30 mm. Composite layup was selected in section assignment. The analysis involved small displacement analysis consisting of a series of vertical displacement-controlled increments applied at the loading points. Timber was modelled as an orthotropic linear-elastic material, for the wood of the strength class CL24h [22, 23]. Results of the FE analysis showed good agreement in the case of both tested series, especially in terms of stiffness (differences 3 %). In the case of maximum load this discrepancy is a bit higher (around 6 % difference). The study is elaborated in [24].

Beside simulations conducted in Abaqus CAE, there are several design-oriented programs that include layered materials such as CLT. In [25], commercial program RFEM in combination with RF-LAMINATE add-on module [26] by Dlubal Software GmbH was selected for the analysis. It is a 3D finite element analysis software for structural analysis. For CLT applications, the Mindlin-Reissner theory of thick orthotropic plates [27] is more relevant than the Kirchhoff theory of thin plates, since the transverse shear deformation cannot be ignored.

CLT panel was modelled as a 2D surface without its composition and material properties. Geometry of the surface and support conditions were entered in accordance with experimental test set-up. FE model of CLT panel in RFEM is presented in Figure 12. The calculation is carried out using the orthotropic material model according to the Mindlin-Reissner plate bending theory. After defining general data for calculation, composition and material properties should be assigned to the CLT panel.

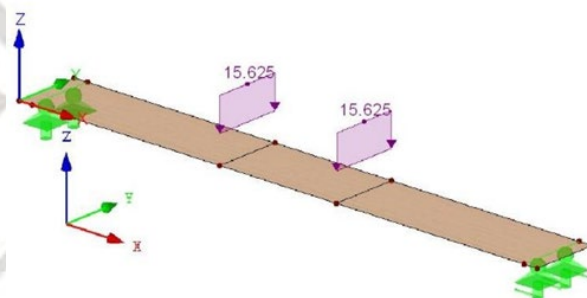


Figure 12. Numerical shell model in RFEM [25]



4.2 Simulations using the original software

Within a project, an efficient model for progressive failure analysis of cross-laminated timber panels under out-of-plane bending load has been developed. To provide the computational efficiency, the model kinematics is based on the full-layerwise plate theory [11] of composite materials, providing 3D stress and strain fields necessary for accurate prediction of damage initiation and progression. Damage initiation and associated failure mode on the lamina level are determined using the 3D Hashin failure criterion [28] with typical strength values of timber material, while the post-failure behaviour is described using a smeared-crack-band model [29, 30] introducing the bilinear strain-softening curves. This allows capturing different timber response in tension (brittle) and compression (ideally-plastic) in the entire considered (3D) domain.

The model bypasses the use of solid finite elements, by employing the layer-wise kinematics based on the Reddy's full-layerwise plate theory of composite materials. It provides the 3D stress and strain field on the lamina level, necessary for accurate prediction of damage initiation and progression.

To support the experimental program, the computational analyses are performed using an original layered finite element framework [2], which is extended within the Substrate4CLT to account for the post-failure behaviour of CLT. Excellent agreement of obtained load-deflection curves and strain distributions verified that the proposed methodology can be efficiently applied in progressive failure analysis of mass timber, but also in other layered beam- and plate-like structures in bending, contributing to the emerging field of computational mechanics of bio-based composite structures.

Besides tailoring the Hashin damage criterion to be applicable for analysis of timber material, the main challenge within the project was to adapt the conventional SCB model to be applicable in the failure prediction of CLT structures. For this purpose, the softening curves in tension and compression are adjusted (Figure 13, right) to capture distinct post-failure behaviours of timber in tension (brittle) and compression (ideally-plastic). The considered damage law assumes that the total released strain energy of the considered domain is equal to the energy needed to create a crack that passes through it. The released strain energy of a failed domain (i.e. finite element) is determined as a product of the area under the stress-strain curve (Figure 3, right) and characteristic domain length.

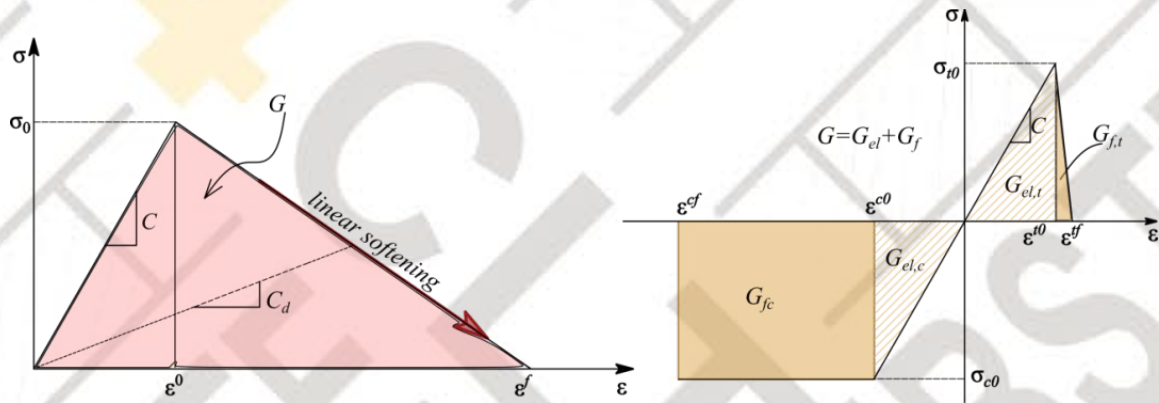


Figure 13. General constitutive relationship with linear softening law applied in smeared crack band damage model (left); improved constitutive relationship for timber with modified softening laws (right)

In the finite element model, geometry, boundary and loading conditions were adopted in accordance with the experimental test setup, defined within the WP2. Whenever possible, due to symmetry in loading and boundary conditions, only a quarter of the CLT panel was modelled to reduce the number of DOFs, with appropriate symmetry constraints.

The stress analysis was performed using only 24×3 Q8 layered quadrilateral elements with reduced integration. Every lamina was divided in two sub-laminas, adopting the linear distribution of displacements along the sub-lamina thickness. In order to avoid stress concentrations, an external load was smeared on subarea of 160×240 mm², according to the experimental test setup. The conducted computational progressive failure analysis showed linear-elastic behaviour until failure, replicating experimental results very closely.



The overall damage pattern predicted using the proposed model is illustrated through a spatial plots of fiber damage variables (Figure 14) at the failure load, associated with the fiber tension ($d_{ft} = 1$). As can be seen from Figure 14, ultimate failure occurred due the FT failure of the bottom longitudinal lamina, which is quite similar to the observation from experiments. RS failure appears in transverse layer, after the load reaches the maximum value and starts dropping, as can be seen through spatial plots of transverse damage variables (Figure 15). This is also in accordance with experimental observation.

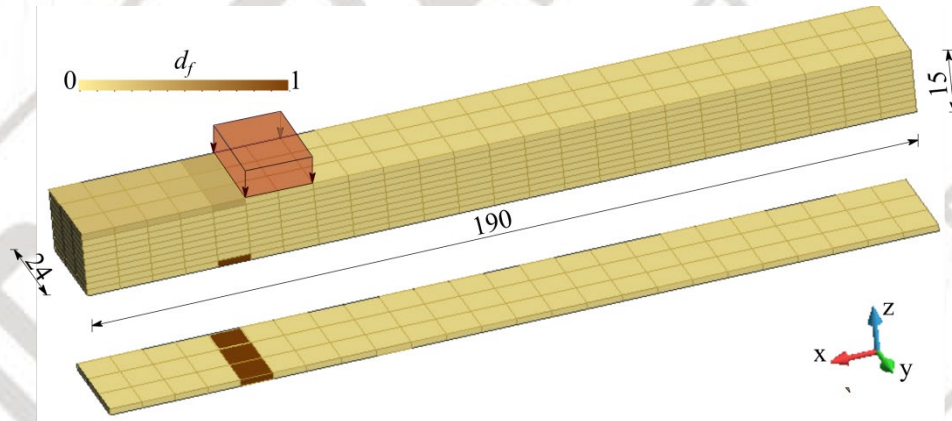


Figure 14. Fiber failure patterns of CLT panel, plotted using the FLWTFEM at the failure load ($d_{ft} = 1$).
The bottom figure illustrates only the bottom layer within a CLT panel

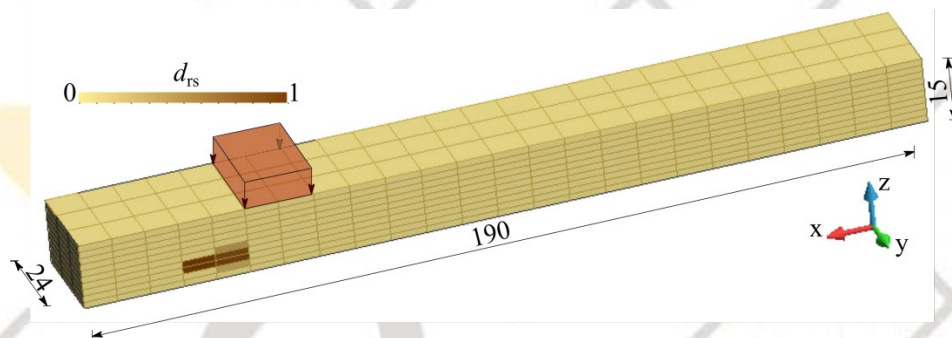


Figure 15. Rolling shear (RS) failure pattern of CLT panel plotted using the FLWTFEM after the load reaches the maximum value and starts dropping

The study within a project also showed that the overall strain distribution predicted by the proposed model is quite similar when compared against the experimental measured strains. Also, the strain values in tension and compression zones were approximately the same at all load levels. Small differences between numerical and experimental results can be justified due the fact that each lamina was homogenized with the average modulus of elasticity, while in reality each lamina is inhomogeneous and the material properties of timber vary. With load increase, no displacement of neutral axis position was recorded, which confirms that wood plastification on the compressed side of the cross-section was limited due to early appearance of cracks in tension zone.

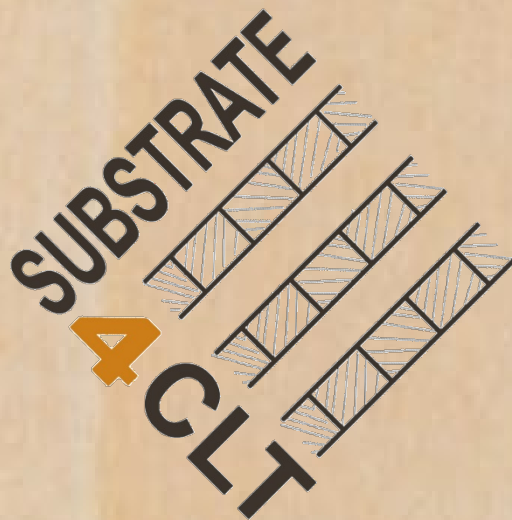


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